Knowledge Spillovers in Cities: An Auction Approach

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ABSTRACT: This paper proposes a micro-foundation for knowledge spillovers. I model a city in which free knowledge transfers are bids by experts to entrepreneurs who auction jobs. These knowledge bids resemble a consultant's pitch to a potential client. Two fundamental properties of knowledge underlie the model: First, it is often necessary to reveal some knowledge to demonstrate its value. Second, knowledge is freely reproducible. Larger cities generate more meetings between experts and entrepreneurs, resulting in more learning and better matches. Larger cities also foster competition for jobs, which motivates experts to raise their knowledge bids. These results demonstrate how competitive behavior can be a source of agglomeration economies, and contribute to explain the higher productivity of urban workers.

Key words: agglomeration, auction, knowledge spillovers.

JEL classification: D44, D83, R23, R39

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1. Introduction

Urban environments have the potential to catalyze learning and networking activity. In Alfred Marshall's influential argument, chance encounters and imitation take place whenever individuals live in close proximity, and knowledge spillovers in cities happen quite inadvertently.[1] Intuitively, however, knowledge spillovers also result from deliberate individual decisions (Helsley and Strange, 2004). The motivation behind these individual decisions remains little understood,[2] in the face of growing evidence on the crucial role that knowledge spillovers play in shaping the geography of production in modern economies.

The main objective of the paper is to explain why people sometimes let their knowledge 'spill', instead of selling it in markets. I model uncompensated knowledge transfers (knowledge spillovers) as bids by experts to entrepreneurs who auction jobs. An expert could be a consultant pitching an idea to a potential client, for instance a free advice on how to advertise a product. Unlike extant literature that focuses on imitation and reciprocity, this explanation for knowledge spillovers relies on competitive behavior and on the fundamental properties of knowledge as an input. I use such knowledge spillovers as a micro-foundation for agglomeration economies, to show how large cities improve knowledge diffusion and offer better job matches between experts and entrepreneurs. Interestingly, heightened competition for jobs enhances experts' willingness to transfer free knowledge, providing a new explanation for the higher productivity of workers in large and dense urban areas.[3]

Interest in urban knowledge diffusion stems from the presumption that density facilitates face-to-face transfers of productive ideas. Patent citations provide direct evidence that physical proximity favors knowledge diffusion, and Jaffe, Trajtenberg, and Henderson (1993) find that a patent holder is more likely to

[1]See Marshall (1920). Glaeser (1999) provides a model formalizing this idea, in which knowledge flows through imitation of the old by the young, with the old getting a share from the young's returns to a successful skill transfer.

[2]Puga (2010), in his review of the literature, argues that: "We have good models of agglomeration through sharing and matching, but not a deep enough understanding of learning in cities."

[3]A vast empirical literature in urban economics establishes that firms and individuals in larger agglomerations are more productive; see Puga (2010) for a review of the literature. The classic explanations for this productivity advantage are so-called 'agglomeration economies'; processes through which interactions between firms or individuals are facilitated in cities, in a way that makes these units more productive.

cite other patents from inventors who are geographically close. The importance of face-to-face interactions is revealed in the very localized nature of production externalities.[4] Rosenthal and Strange (2008) and Arzaghi and Henderson (2008) find that the productivity gains from spatial proximity to other people decay sharply with distance. This decay suggests that face-to-face interactions, whose cost is especially sensitive to distance, are prominent among the factors making people in larger and denser cities more productive.

Wage regressions corroborate the importance of learning in cities. Workers in large cities earn on average 30% more than workers in rural areas (Glaeser, 2011), but this premium does not immediately accrue to a worker upon moving to a bigger city. Much of these earning gains arise only as a worker accumulates experience in the city (Glaeser and Maré, 2001, De la Roca and Puga, 2013). Recent estimates from Spain show that about half of the earning advantage of urban workers is acquired through time (De la Roca and Puga, 2013), which suggests that learning plays a central role in increasing urban workers' productivity. Jacobs (1968), Lucas (1988), and others take the argument further and conclude that in a knowledge-based economy, idea transfers in cities are fundamental to economic growth.

I develop a model in which individuals meet in a city, and then start to work. During a meeting, an expert chooses how much knowledge to freely transfer to an entrepreneur. After learning from all experts, an entrepreneur hires one of them and produces a good with a constant returns to scale technology, using both the knowledge obtained during meetings and that acquired from the hired expert. The model's intellectual foundation and defining assumptions – that the value of knowledge is unobservable and communicated at no cost – derive from two key ideas about the properties of knowledge as an input. The first idea, from Arrow (1962), is that one often must possess knowledge to assess its value, or reveal an idea to demonstrate its usefulness. This is unlike a physical object like a car, which can be taken for a test drive and brought back. The second idea is that

[4]Few studies are able to directly assess the importance of knowledge spillovers relative to other agglomeration forces. Rosenthal and Strange (2001) find evidence supporting all three sources of agglomeration identified by Marshall (1920) - input sharing, labor pooling and knowledge spillovers - with labor pooling being the best predictor of industry agglomeration. Ellison, Glaeser, and Kerr (2010) also find evidence for all three sources by looking at coagglomeration patterns for different industries, but conclude that input sharing (specifically, proximity to suppliers and customers) has the most explanatory power.

knowledge is freely reproducible, or has a negligible marginal cost of production once a blue-print exists.[5]

Arrow's property suggests the motivation behind uncompensated knowledge transfers. For an expert, a free transfer is a bid for a job contract, a means to prove how knowledgeable she is. For an entrepreneur, who auctions a job to experts, the free knowledge transfers are an opportunity to accumulate knowledge, and to find the best expert for a particular job. An auction framework delineates the strategic incentives to give some valuable knowledge without direct payment, but not all. It identifies conditions under which knowledge 'spills' in non-market interactions, as opposed to market transactions occurring after the best expert for a job is revealed and starts working for an entrepreneur. Agglomeration economies result from free reproducibility of knowledge and from the greater number of meetings between experts and entrepreneurs in a larger city. The model uncovers a new agglomeration force, stemming from competitive behavior: As the number of experts competing for jobs in a city increases, their willingness to freely transfer knowledge – the size of their knowledge bid – also increases. Dividing agglomeration forces into sharing, matching and learning as in Duranton and Puga (2004), larger cities in this model stimulate more learning and better matches.

Auction theory offers intuitive models of people's behavior in competitive environments, which reward individuals for being smarter and more knowledgeable than others.[6] As such, the model evokes so-called 'industrial clusters ,' in which networking is essential, job changes are frequent and, as shown in Freedman (2008), workers initially accept lower wages while they develop their reputations. The model also captures the networking behavior of workers in finance, advertising, law, graphic design and other consulting industries, as they 'pitch' their ideas to potential clients.[7]

[5]The first model relating increasing returns to the free reproducibility of knowledge is from Romer (1990).

[6]At another extreme, Niehaus (2011) assumes that people transfer knowledge non-strategically, as long as the benefits to the learner exceed the costs to the sender.

[7]While Marshall (1920) emphasizes within-industry externalities, Jacobs (1968) highlights the importance of cross-industry spillovers. Glaeser, Kallal, Scheinkman, and Shleifer's (1992) finding that a diverse industry mix is more conducive to industry growth supports the preeminence of Jacobian externalities. Henderson, Kunkoro, and Turner (1995), however, find that Marshallian externalities are more important, at least for mature industries.

The paper contributes to several strands of literature in urban economics, innovation, knowledge markets and mechanism design (auctions). The innovation literature, unlike this paper, emphasizes the role of reciprocity in fostering incentives to engage in free knowledge transfers. In a seminal paper, von Hippel (1987) uses survey evidence to document how the expectation of reciprocal transfers motivates engineers in the steel industry to share knowledge with their peers. With the exception of Glaeser (1999), who focuses on imitation, reciprocity is also the motivation for knowledge transfers in theoretical models of knowledge diffusion and creation. Jovanovic and Rob (1989) propose a model in which agents can develop ideas privately, but can also share their ideas with other agents, such interactions leading both to imitation (diffusion) and invention (creation). Berliant and co-authors (2006, 2008 and 2009) refine Jovanovic and Rob's insight that new and better ideas originate from the contact of different ideas, and provide an analysis of the costs and benefits of urban diversity for knowledge creation. Helsley and Strange's (2004) model is the first in which the choice of how much knowledge to transfer is endogenous. In their model, agents living in a city barter knowledge and develop a reputation for cooperation, in line with von Hippel's evidence on 'know-how' sharing. Finally, Davis and Dingel (2012) consider idea exchange as an agglomeration force in a system of cities model.

The expectation of reciprocity certainly motivates many free knowledge transfers, but my model purposely excludes the possibility of sharing or reciprocity. I provide an alternative explanation for uncompensated knowledge transfers, based on the incentives to communicate how knowledgeable one is. Supporting this explanation, Appleyard (1996) finds that reciprocity is less important in high-tech or other rapidly evolving industries, precisely the kind of industries in which agglomeration economies are strongest (Moretti (2004), Fu (2007), and others). Ignoring reciprocity reconciles theories on knowledge spillovers with these empirical findings.

The paper also relates to the literature on knowledge markets. Bhattacharya and Ritter (1983), Anton and Yao (2002), and others draw from Spence's (1973, 1974) work on market signaling to show that Arrow's property can lead firms to provide a 'voluntary disclosure of knowledge' (another expression for 'free knowledge transfers'). My model differs from existing models of voluntary knowledge disclosure, in that it defines a free knowledge transfer not as a 'signal', but rather as a 'bid', a simpler concept that is better suited to informal and semi-formal interactions in competitive urban environments.

Finally, the paper relates to the literature on auction theory. The solution to the basic version of the model only involves the simplest case of a first-price auction with no entry cost. However, knowledge auctions in an urban environment differ in two ways from existing auction models. First, a knowledge auction is all-pay from the auctioneer's perspective, but not from that of the bidder. Second, entry costs and search costs in urban environments are endogenous and depend on city population.

The remainder of the paper is organized as follows: Section 2 presents the basic knowledge auction model in a closed city. Section 3 is an extension of the basic model in which experts and entrepreneurs face a binding time constraint. Section 4 analyzes the equilibrium size and composition of an open city. Section 5 provides a discussion of the results and Section 6 concludes.

2. A model of knowledge transfers in a closed city

Consider a closed city with N inhabitants, a number X of which are experts who hold some knowledge, and a number E of which are entrepreneurs who can use that knowledge in production, so that $X + E = N$.

Experts and entrepreneurs play a two-stage game: first a meeting stage, then a production stage. At the meeting stage, experts learn their type and choose how much knowledge to freely transfer to entrepreneurs that they meet in the city. Entrepreneurs learn from the knowledge transferred by experts. At the production stage, each entrepreneur hires one expert and pays her for the knowledge that she has not already given at the meeting stage.[8] Entrepreneurs then produce a consumption good, using both the knowledge accumulated at the meeting stage and that acquired at the production stage of the game.

Let i index the set of X experts, and j index the set of E entrepreneurs. The value of expert i's knowledge to entrepreneur j is denoted by k_{ij}, so expert i's type is a E-vector of knowledge $k_i = (k_{i1}...,k_{ij},...k_{iE})$. k_{ij} is independently and identically distributed over the [0,1] uniform distribution, for all $i \in \{1,...,X\}$

[8]Online Appendix F solves a version of the model in which an entrepreneur can hire many experts, and provides numerical evidence that an entrepreneur prefers to hire just one.

and $j \in \{1,...,E\}$. The value of an expert's knowledge to an entrepreneur is independent of its value to another entrepreneur, to reflect the idea that different experts are proficient in different fields, and that each entrepreneur puts a distinct value on any expert's knowledge.[9] Knowledge has two properties: there is no cost of communicating it (free reproducibility), and the value of an expert's knowledge is unobservable to entrepreneurs or other experts. So there is no cost of meeting, an assumption that I relax in Section 3. The meeting technology is such that every expert meets every entrepreneur in the first stage of the game, so the number of meetings is $E \times X$. The key assumption is that larger cities feature more meetings per capita. The particular form of the matching function that realizes all possible meetings only brings the benefit of simplicity, and it is not necessary to derive the model's main results. Section 3 proposes a microfoundation for a matching function that delivers more meeting per capita as city size increase.

Define $b_{ij}(k_{ij},X)$ as the knowledge transferred for free by expert i to an entrepreneur j that she meets in the first stage of the game. The notation anticipates that this transfer will be function of k_{ij} and X. Each entrepreneur uses the same constant returns to scale, additive production function to produce a single good y, with knowledge as the only input. Entrepreneur j produces an amount y_j of the good, which is equal to the knowledge received for free at the meeting stage of the game, plus the knowledge paid for at the production stage (denoted by x_j), so that:

$$y_j = \left(\sum_{i=1}^{X} b_{ij}(k_{ij},X) \right) + x_j. \tag{1}$$

The utility function of both experts and entrepreneurs is $u(y) = y$, where y is the amount of the good consumed. Note that perfect substitutability of experts' knowledge is not a necessary assumption, and online Appendix D numerically replicates the model's key results using a Cobb-Douglas production function.[10]

The wage setting mechanism is such that an expert earns a wage equal to his

[9] This independence does not affect any of the results in this section.

[10] With Cobb-Douglas production, there is some complementarity between the knowledge of different experts, and the model is not simply one of knowledge diffusion. A production function with complementarities captures some features of knowledge creation, because entrepreneurs combine the knowledge of different experts to produce something different from the sum of its parts.

productivity at work, x_j, paid for in units of the good.[11] This wage contract depends on the actual surplus created by an expert, not on the belief of an entrepreneur about her type. Given the CRS, additive production function, it follows that expert i hired by entrepreneur j earns a wage equal to $k_{ij} - b_{ij}(k_{ij}, X)$; the value of the knowledge that she has not already given during the meeting stage of the game. Knowledge transferred for free cannot be given back and as such is never paid for. An expert can work for any number of entrepreneurs, and her wage is the same as her utility from obtaining a job, because of the linear utility function.

It only remains to specify the job allocation mechanism. For an entrepreneur, the unobservability of experts' knowledge presents an opportunity to ask for free knowledge, with a job offered as a reward. A particularly attractive mechanism is the simple rule awarding a job to the expert who transfers the most knowledge during the meeting stage of the game. As the next section demonstrates, this rule (which turns out to be a first-price auction) allows an entrepreneur to find the best expert for a particular job, as well as to obtain free knowledge from all other experts.[12] This first-price auction is arbitrarily close to any optimal mechanism as the number of experts becomes large, in the sense that an entrepreneur freely uses the knowledge of all experts. In the online appendix, I provide evidence that this job allocation mechanism is optimal under two important extensions of an entrepreneur's action space. First, online Appendix E allows entrepreneurs to require a minimum knowledge transfer, and shows that they prefer to set it to zero. This result distinguishes knowledge auctions from standard auctions in which an auctioneer can increase its expected payoff by setting a positive reservation price (Myerson, 1981).[13] Second, online Appendix F allows entrepreneurs to hire any number of the top experts - who transferred the most knowledge

[11] Equivalently, the wage is settled through Nash bargaining over the surplus produced by a working expert, and λ, the bargaining power of an entrepreneur, is equal to zero. A different value of λ would not affect any of the main results of this section.

[12] A commitment problem arises if an entrepreneur has an incentive to motivate knowledge transfers by pretending ex-ante that he will hire only one expert, but prefers ex-post to hire many experts to earn some surplus from their work. Assuming that entrepreneurs do not get a share of the experts' surplus ($\lambda = 0$) resolves that commitment problem. Therefore, entrepreneurs in the model do not need to commit to hiring only one expert in a Bayesian equilibrium.

[13] In a normal auction model like that of Myerson (1981), the seller only receives the largest bid. A knowledge auction, however, is all-pay from the perspective of an entrepreneur, so a reservation price is not desirable because it deters low type experts from participating in the auction.

during meetings - and finds that they prefer to hire only one expert (this last result, however, is only supported by numerical evidence). Of course, there exist plausible extensions of an entrepreneur's action space under which the first-price auction is not optimal, for instance if entrepreneurs can make additional transfer payments beyond the wage setting mechanism.[14] The first-price auction, however, is probably a more realistic representation of entrepreneurs' behavior in informal or semi-formal urban meetings than the richer incentive schemes suggested by optimal mechanism design with larger action spaces.

2.1 Results

The previous section defined the job allocation mechanism (an auction) and the production stage of the game, so it is now possible to work backwards and solve for an expert's equilibrium strategy at the meeting stage of the game. Using auction terminology, k_{ij} becomes expert i's valuation for a job with entrepreneur j, and $b_{ij}(k_{ij},X)$ becomes her bid for a job. An expert optimally works for every entrepreneur who offers her a job, and the E auctions that she participates in are strategically independent. The strategy of an expected utility maximizing expert i, when meeting entrepreneur j, is to transfer the amount of free knowledge that maximizes the probability that she gets the job times the utility (wage) that she obtains if she is hired. If all other experts use the same bidding function b, i.e. in a symmetric equilibrium, then the problem of an expert of type $k_{ij} \in [0,1]$ is to submit a bid $b_{ij} \geq 0$ that solves:

$$\max_{b_{ij}} prob\left(b_{ij} > \max_{l \in \{1,\dots,X\}\setminus\{i\}} b(k_{lj},X)\right) \times (k_{ij} - b_{ij})|_{b(k_{ij})=b_{ij}}. \quad (2)$$

Equation (2) demonstrates that for an expert, a meeting is equivalent to participation in a first-price auction. The highest bidder wins and 'pays' his bid, because an expert cannot be paid at the production stage for the knowledge already transferred at the meeting stage. The solution to an expert's problem is well-known, as $b(k_{ij},X) = ((X-1)/X)k_{ij}$ is the unique Nash equilibrium of a first-price auction in the independent private value model with quasi-linear utility

[14] Appendix B shows that entrepreneurs can increase their expected utility by offering a wage top-up to an expert winning the auction.

and a $U[0,1]$ distribution of valuation, see for instance Jehle and Reny (2000).[15] I dropped the i and j indices because of symmetry and independence across auctions. Note that an expert has no incentives to make a direct monetary payment (i.e. a payment in units of the good) to an entrepreneur, because experts always have enough knowledge to 'pay' their bid. Knowledge is freely reproducible, so bidding knowledge is always preferable to bidding money. Lemma 1 summarizes these results:

Lemma 1 *For an expert, a meeting is equivalent to participation in a first-price auction that requires solving equation* (2)*. There exists a unique symmetric Bayesian Nash equilibrium bidding function that is strictly increasing in k and with $b(0,X) = 0$, given by $b(k,X) = \frac{X-1}{X}k$.*[16]

An important corollary of Lemma 1 is that each entrepreneur k hires the expert that is best for him, i.e. with the highest knowledge value k_{ij} from the set $\{k_{1j},\ldots,k_{Xj}\}$. This is a consequence of a bidding function that strictly increases in k, for all $X > 0$. Another key property of the bidding function is that $\frac{\partial b}{\partial X} > 0$, for all $k \in [0,1]$ and all $X > 0$ (I remove subscripts when no confusion if possible). That is, the value of the free knowledge transferred by each type of experts increases with the number of other experts competing for jobs in the city. Before stating and proving the main proposition of this section on total production in a closed city, I derive a number of intermediate results.

To compute $\mathbb{E}(x)$, the expected wage of an expert, define a random variable $k_{(X,X)}$ representing the highest order statistic of X draws from the *i.i.d.* $U[0,1]$ distribution, with probability distribution function $f(k_{(X,X)}) = Xk_{(X,X)}^{X-1}$. The expected wage is then:

$$\begin{aligned} \mathbb{E}(x) &= \int_0^1 \left(k_{(X,X)} - b(k_{(X,X)},X)\right) f(k_{(X,X)})dk_{(X,X)} \\ &= \int_0^1 \left(k_{(X,X)} - \frac{X-1}{X}k_{(X,X)}\right) Xk_{(X,X)}^{X-1}dk_{(X,X)} \end{aligned} \quad (3)$$

[15] The result holds for all quasi-linear utility functions, so considering Nash bargaining for wages with λ as the bargaining power of an entrepreneur would not affect the bidding function of experts, which stays the same for wages equal to $(1-\lambda) \times (k - b(k))$ and $0 \leq \lambda < 1$.

[16] It would be possible to show that this equilibrium is unique, without restricting attention to symmetric equilibria with bidding function strictly increasing in k and $b(0,X) = 0$. See Maskin and Riley (2003) for a proof whose assumptions are satisfied by the distribution of valuation and utility specification that I use (they make the additional assumption that experts never bid more than their valuation).

$$= \frac{1}{X+1},$$

where the first line is simply the expected value of the best expert's knowledge (highest order statistics) minus her expected bid, and the second equality substitutes for the bidding function.

The expected utility of an expert can be computed both before and after she learns her type (but note that an expert always knows her type when she bids). Such distinction is useful in extensions of the models in which experts can migrate to an open city, covered in Section 4. The expected utility of an expert i after learning her type k_i is equal to:

$$\begin{aligned} &\mathbb{E}\left(\sum_{j=1}^{E}(k_{ij}-b(k_{ij},X))\,\mathbb{I}\left[k_{ij}>k_{hj},\forall h\in\{1,...,X\}\backslash\{i\}\right]\right) \qquad (4)\\ &=\sum_{j=1}^{E}k_{ij}^{X-1}\left(k_{ij}-b(k_{ij},X)\right)\\ &=\sum_{j=1}^{E}\frac{k_{ij}^{X}}{X}, \end{aligned}$$

where $\mathbb{I}\{\cdot\}$ is an indicator function equals to 1 if expert i wins a job from entrepreneur j. The first equality obtain because $\mathbb{E}$ is a linear operator, and k_{ij}^{X-1} is the probability that expert i wins the auction of entrepreneur j. The second equality comes from substituting the bidding function. Equation (4) shows that after learning her type, the expected utility of an expert increases with the number of entrepreneurs and with the value of her knowledge, and decreases with the number of competing experts.

The expected utility of an expert before learning her type is equal to:

$$\frac{1}{1+X}\times\frac{E}{X}, \qquad (5)$$

the expected wage from equation (3), times E/X, the number of jobs that each expert can expect to win. Therefore, the expected utility of an expert before she learns her type increases with the number of entrepreneurs and decreases with the number of other experts.

To find the expected utility of an entrepreneur, recall that they do not get a share of the surplus produced by working experts. So an entrepreneur j's expected utility is equal to the expected value of the knowledge transferred during meetings:

$$\mathbb{E}\left(\sum_{i=1}^{X} b(k_{ij},X)\right) = X\int_0^1 b(y,X)dy = X\int_0^1 \frac{X-1}{X} y dy = \frac{X-1}{2}. \tag{6}$$

This function displays increasing returns to the number of experts. Increasing returns arise because of the competitive behavior of experts, who bid more as the number of other experts increases, and because of the special nature of knowledge. That is, the knowledge auction is *all-pay* from the perspective of an entrepreneur, who makes productive use of the knowledge bids of all experts, including those that he does not hire. The expected utility of an entrepreneur does not depend on E, as there is no competition between entrepreneurs in the model.

Finally, an expression for expected total production in the city results from simple arithmetics.

Proposition 1 *Expected total production in a closed city is equal to* $Y = E\left(\frac{X-1}{2} + \frac{1}{X+1}\right)$

Proof The term inside the bracket is expected production per entrepreneur, which is equal to the expected sum of the knowledge transfered to each entrepreneur in the meeting stage of the game (equal to $(X-1)/2$, from equation 6), plus the expected knowledge obtained by each entrepreneur during the production stage (equal to $1/(X+1)$, the expected wage paid by an entrepreneur, from equation 3). All entrepreneurs are identical, so expected total production is equal to E times per entrepreneur production. □

The defining feature of total expected production is the presence of external returns to scale. That is, doubling the number of experts and entrepreneurs in the city more than doubles Y, despite the production function itself having constant returns to scale. There are two sources of increasing returns to the number of city inhabitants: an increase in the number of meetings (extensive margin) and an increase in the productivity of each meeting (intensive margin).

First, increasing returns at the extensive margin come from the matching function, which displays increasing returns to scale because it matches each entrepreneur with each expert. This is apparent in the expression for Y in Proposition 1. This expression is equal to the number of entrepreneurs times the sum of the knowledge transferred in both stages of the game by all experts, and it closely resembles the expression for the number of meetings ($E \times X$). Second, increasing returns at the intensive margin originate from experts' choice to reveal more knowledge during meetings as competition for jobs heats up. This theoretical prediction is supported by robust empirical evidence that bids in independent private value first-price auctions increase with the number of participants (Kagel, 1995). This intensive margin is also apparent in the expression for Y; the average knowledge transferred in both stages by an expert, $\left(\frac{X-1}{2} + \frac{1}{X+1}\right)$, increases more than proportionally with the number of experts.[17] All the increasing returns come from $\frac{X-1}{2}$, the average knowledge transferred during meetings. At the intensive margin, there are only increasing returns to X, the number of experts in the city. These returns vanish as X becomes large and experts bid the entire value of their knowledge at the meeting stage ($\lim_{X\to\infty} b(k) = \lim_{X\to\infty}((X-1)/X)k = k$). In this case, the auction mechanism is necessarily optimal for entrepreneurs, who extract all the knowledge of every expert during meetings. The city is also efficient, as expected production Y tends to its maximum level – the expected total amount of knowledge in the economy – of $(E \times X)/2$.

Intuitively, increasing returns to city size arise because interactions are cheaper and the number of potential partners is higher in larger cities. The model captures this intuition through a matching function such that each expert and entrepreneur in the city meets once, at no cost. Unsurprisingly, increasing returns at the extensive margin is a prerequisite for increasing returns at the intensive margin, and heightened competition or better matches in larger cities depend on increasing returns from the matching function. This paper formalizes and extend these ideas by combining a theory of strategic interaction – auction theory – with the fundamental properties of knowledge as an input. The idea that valuable knowledge must be given in order to demonstrate its value explains the incentive of an expert to transfer knowledge. Free reproducibility in turn leads to increasing returns, because each expert is able to transfer his knowledge to many entrepreneurs without losing it. In some sense, Arrow's property enhances the efficiency of

[17] $\frac{d\frac{1}{X}\left(\frac{X-1}{2}+\frac{1}{X+1}\right)}{dX} = \frac{X^2+4-4X}{2X^2(X-2)^2} - \frac{1+2X}{X^2(X+1)^2} = \frac{1}{2X^2(X+1)^2}\left(X^2 - 2X - 1\right) > 0$ for all integers $X > 2$. There are no increasing returns from competition when starting from $X = 1$, because if there is only one expert in the city she bids nothing but is hired by everyone, which is efficient compared to the case in which there are two experts who bid against each other, but with only one of them working.

cities. The unobservability of experts' knowledge empowers entrepreneurs to ask for free transfers during meetings. This diffusion of knowledge might not happen if the value of experts' knowledge was perfectly observable.

Another defining property of the knowledge auction lies in its ability to match each entrepreneur with the expert whose knowledge is most valuable to him. The value of an expert's knowledge is uncorrelated across entrepreneurs, so different experts, with expertise in different fields, are matched with different entrepreneurs. This is consistent with the assumption that an expert's type is not common knowledge (as it probably would be if the same expert was best for every entrepreneur) and that meetings are necessary to reveal the nature and extent of expert's knowledge. In fact, the potential for better matches between workers and firms is believed to be one of the main agglomeration forces generating cities.[18] The knowledge transfer mechanism in this paper provides a possible explanation for how entrepreneurs manage to identify more suitable workers when the pool of candidates is larger. In bigger cities, more numerous and valuable informal meetings generate more productive working relationships.[19]

To summarize, the auction framework suggests answers to the why, how and when of uncompensated knowledge transfers. For an expert, meeting an entrepreneur is an opportunity to display the extent of his knowledge, in a bid to impress an entrepreneur enough to get a job. For an entrepreneur, meeting an expert is an opportunity to receive free knowledge, and to find the best expert for a job. Market transactions, in the form of compensated knowledge transfers at the production stage of the game, only follow non-market interactions after revelation of the best expert.

[18]See Duranton and Puga (2004) for a review of the literature on micro-foundation for agglomeration economies

[19]The analysis of these gains is hard within the model because increasing the productivity of meeting decreases productivity at work. Note, however, that an entrepreneur gains from better matches under the assumption that he captures a share λ of a hired expert's production at work. In equilibrium, an entrepreneur can infer the type of an expert from his bid, and it is easy to show that the expected production of the best expert is larger than that of the second best. Under the assumption that $\lambda = 0$, however, 'hiring the best expert' is a good strategy for an entrepreneur only because it provides incentives for experts to transfer knowledge.

3. Adding a time constraint

This section introduces a time constraint in the model of the previous section, to relax the assumption that each expert meets each entrepreneur in the city. Experts incur a time cost of searching for, meeting with, and working for entrepreneurs, of whom they meet only a random subset. The search cost of an expert decreases with the number of entrepreneurs in the city. Such a decrease in the cost of meeting other people is widely acknowledged as a fundamental advantage of modern cities (e.g. Glaeser and Kohlhas, 2004).[20] As a result, the matching function receives a microfoundation that delivers more meetings per capita in larger cities. So just as in the model of Section 2, increasing returns arise because larger cities enable more meetings per capita, and heightened competition renders each meetings more productive. At the end of this section, I discuss an alternative model with constant search cost, in which experts are able to sort into meetings with entrepreneurs for whom their knowledge is most valuable. I solve this model in online Appendix H.

The city population is $X + E = N$, with $z = E/X$ defined as the ratio of entrepreneurs to experts in the city. In what follows I hold city composition z constant, and I explore the impact of an increase in city size N. Each expert and each entrepreneur has one unit of time. The time cost of meeting to give or receive knowledge is the same for experts and entrepreneurs, and equal to $c < 1$. An expert who works for an entrepreneur incurs an additional time cost equal to c. Finally, an expert who wishes to find an entrepreneur to meet with incurs a search cost equal to $c(E)$, with $c(1) = c$, $\lim_{E\to\infty} c(E) = 0$ and $c'(E) < 0$. This last property is crucial and captures the idea that the search cost is lower when entrepreneurs are numerous. The set of entrepreneurs that each expert searches for is random (see online Appendix G for a model in which an expert's searches are directed towards entrepreneurs for whom her knowledge is most valuable.)

I make two assumptions on the time constraint of experts and entrepreneurs. First, the time constraint need only bind in expected value. That is, an expert

[20]This assumption can be consistent with a cost of living that increases in city population. For instance, individuals could first incur a cost of housing and of commuting to a central business district, both of which increase with population. Once in the CBD, however, workers find it cheaper to search for a meeting partner in a large city. For simplicity, I do not model commute and housing costs in this section.

chooses a number of searches before learning her type, such that her expected use of time is equal to 1. Second, I restrict attention to cases in which the time constraint of an expert binds, but that of an entrepreneur does not. It will be easy to show that this is always the case if N is large enough and if $z > 1$, i.e. if there are more entrepreneurs than experts.[21] Together, these assumptions imply that an expert's searches are always successful, because an entrepreneur has time to meet with every expert who searches for him.

3.1 Results

I first solve for S, the number of searches that each expert makes. It is always optimal for experts to reach their boundary restriction, so the time constraint of an expert in expected value can be written as:

$$c(E)S + c(1)S + c(1)\frac{E}{X}\left(1 - \left(1 - \frac{S}{E}\right)^{X}\right) = 1, \tag{7}$$

where the third term, equal to $c(1)$ times the expected number of jobs that each experts gets, is derived using the symmetry of the problem and the probability $\left(1 - \frac{S}{E}\right)^{E/z}$ that an entrepreneur meets no expert and does not allocate a job.[22]

In this model, the decreasing search cost implies a greater number of searches per expert as city population increases ($S'(E) > 0$), a result that I state in the following lemma. The lemma also provides a condition on $c'(E)$ ensuring that $S'(E)$ is large enough. This condition will turn out to be sufficient to prove that the bid of each type of experts increases with city size.

[21]The case of N small, in which neither the time constraint of an expert nor that of an entrepreneur binds, is equivalent to the model in Section 2. One argument to support the restriction $z > 1$ comes from the next section, which shows, in a simpler setting without time constraints, that z has to be much larger than 1 to equalize experts' and entrepreneurs' utility in an open city equilibrium, because experts provide an uncompensated externality. If the constraint of an entrepreneur was to bind, then one would need to specify a mechanism matching experts to entrepreneurs, or else there is an inefficiency in large cities as the searches of experts are wasted on entrepreneurs who have no more time to meet.

[22]This coordination friction alone would induce experts to make slightly more searches as city size increases, but this is a very small effect that is not the source of increasing returns in the model. See Rogerson, Shimer, and Wright (2005) for a discussion of such coordination frictions in the search-theoretic literature, in which the type of matching function in my model is often referred to as an 'urn-ball matching process'.

Lemma 2 *If $c'(E) < 0$, then $S'(E) > 0$. Moreover, if $c'(E) < \frac{-2}{E^2}$ then $S'(E) > \left(\frac{S(E)}{E}\right)^2$.*[23]

Proof In Appendix A. □

To show that the expected number of auction participants, denoted by Q, also increases with city size, define $q = S(E)/E$ as the probability that an expert participates in any entrepreneur's auction. The number of experts participating in an auction is therefore a binomial random variable with distribution $B_{X,q}$ and expected value Q such that $Q = qX = \frac{XS}{E} = \frac{S}{z}$.[24] Note that Q, S and q are endogenous variables, but I often drop their dependence on E and X from the notation to improve readability.

To solve for the bidding function of an expert, I follow Harstad, Kagel, and Levin (1990), who study the equilibrium of first-price auctions with an uncertain number of bidders. From the perspective of an auctioneer, the binomial probability that a bidders participate in the auction is:

$$s(a) := prob(a \text{ bidders}) = \binom{X}{a} q^a (1-q)^{X-a}. \tag{8}$$

From the perspective of an expert who already participates in the auction, the binomial probability of a participants is:

$$p(a) := prob(a \text{ bidders}|1 \text{ is a bidder}) = \frac{as(a)}{\sum_{i=1}^{X} is(i)} = \frac{a}{Q}s(a). \tag{9}$$

As before, the value of a job with entrepreneur j for expert i is k_{ij}, which is independently and identically distributed over the [0,1] uniform distribution, for all $i \in \{1,...,X\}$ and $j \in \{1,...,E\}$. If all other experts use the same bidding

[23]One search cost function that satisfies this condition - along with all other assumptions on the search cost - for all E large enough is $c(E) = cE^{-\alpha}$ for $0 < \alpha < 1$.

[24]It is now possible to derive the condition under which the time constraint of an entrepreneur does not bind. In expected value, an entrepreneur's constraint is $c(1)Q \leq 1$, which can be written as $c(1)\frac{S}{z} \leq 1$. The largest value of S occurs at $c(E) = 0$ (i.e. when $N \to \infty$), at which, using equation (7), we find $S = \frac{1-c(1)z(1-e^{-S/z})}{c(1)}$. Plugging this into the constraint of an entrepreneur, we find $z \geq \frac{1}{1+c(1)(1-e^{-S/z})}$. The right-hand side of this expression is always smaller than 1, because $1 - e^{-S/z} > 0$ for any positive values of S or z. So the constraint of an entrepreneur never binds if $z > 1$, even for N arbitrarily large.

function b, i.e. in a symmetric equilibrium, then the problem of an expert of type $k_{ij} \in [0,1]$ is to submit a bid $b_{ij} \geq 0$ that solves:

$$\max_{b_{ij}} \sum_{a=1}^{X} p(a) Prob\left(b_{ij} > \max_{l \in \{1,\ldots,a\} \setminus \{i\}} b(k_{lj})\right)(k_{ij} - b_{ij})|_{b(k_{ij})=b_{ij}} \tag{10}$$

for all $j \in \{1,\ldots,S\}$. In what follows I drop the dependence of b on the model's parameter X, c and z from the notation, and by symmetry the i and j indices are superfluous, so I use $b(k)$ to denote the bid of an expert with value of knowledge k. The following result is a simple adaption of Harstad *et al.* (1990) to the case of the i.i.d. $U[0,1]$ distribution of valuation:

Lemma 3 *For an expert, a meeting is equivalent to participation in a first-price auction with an uncertain number of bidders that requires solving equation* (10). *There exists a unique symmetric Bayesian Nash equilibrium bidding function that is strictly increasing in k, and with $b(0) = 0$, given by:*

$$b(k) = \sum_{a=1}^{X} w_a(k) \frac{a-1}{a} k, \tag{11}$$

where $w_a(k) = \frac{k^{a-1}p(a)}{\sum_{i=1}^{X} k^{i-1}p(i)}$.

Proof See Theorem 1 in Harstad *et al.* (1990). Also, online Appendix G contains derivation of this bidding function in a more general case. □

Note that $b(k)$ is a weighted sum of terms $\frac{a-1}{a}k$, each equivalent to the bid of an expert facing $a-1$ competitors with certainty (see Section 2). The next lemma shows that the bid of each type of experts increases with city population under a sufficient condition that $c(E)$ decrease fast enough with E.

Lemma 4 *If $c'(E) < \frac{-2}{E^2}$, then $b(k)$ is increasing in N, for all $k \in (0,1]$.*

Proof In Appendix A. □

As N increases - holding z constant - the search cost $c(E)$ decreases, and the expected number of auction participants increases. Heightened competition, as in Section 2, leads to greater willingness to transfer free knowledge. The essence of the proof is to show that the probability distribution given by w_a shifts mass to the right as N increases. The bidding function is an expected value of increasing outcomes $\frac{a-1}{a}k$ over this distribution, and thus is increasing in N as well.

To obtain an expression for expected total production in the city, I first compute the expected total knowledge transferred at the meeting stage as:

$$E\left[\sum_{j=1}^{E}\sum_{i=1}^{X} b(k_{ij})\right] = E\sum_{a=1}^{X} s(a)\, a \int_0^1 b(y)\mathrm{d}y$$
$$= EQ\int_0^1 b(y)\mathrm{d}y. \quad (12)$$

Next I compute the expected total knowledge transferred at the production stage, which is equal to the number of entrepreneurs times the expected knowledge transferred at work by an expert who wins an auction:

$$E\sum_{a=1}^{X} s(a)\int_0^1 (y-b(y))ay^{a-1}\mathrm{d}y, \quad (13)$$

where ay^{a-1} is the probability distribution function of the highest order statistic out of a draws from the $i.i.d.$ $U[0,1]$ distribution.[25]

As in Section 2, total expected production in the city is simply a sum of the expected total knowledge transferred at the meeting stage (equation 12) and at the production stage (equation 13). The next proposition is the key result of this section, as it shows that expected total production displays increasing returns to scale even in the presence of a time constraint. That is, if city population doubles, then expected total production more than doubles, so expected per capita production increases.

Proposition 2 *Expected total production is given by:*

$$Y(N) = E\sum_{a=1}^{X} s(a)\int_0^1 (y-b(y))ay^{a-1}\mathrm{d}y + EQ\int_0^1 b(y)\mathrm{d}y. \quad (14)$$

If $c'(E) < \frac{-2}{E^2}$, then $Y(N)$ exhibits increasing returns to scale.

Proof In Appendix A. □

[25]The sum in equation (12) and (13) runs from $a=1$ instead of $a=0$, to highlight the fact that nothing is produced and no knowledge is transferred when there are no auction participants.

As in the model without a time constraint of Section 2, there are two sources of increasing returns to the number of city inhabitants: an increase in the number of meetings (extensive margin) and an increase in the productivity of each meeting (intensive margin). Increasing returns originate entirely from knowledge exchanged in meetings (the term $EQ\int_0^1 b(y)\mathrm{d}y$) in equation 14). At the extensive margin, the expected number Q of experts meeting each entrepreneur increases with N, so larger cities feature more meetings per capita because of declining search costs. Again, the intensive margin derives from the extensive margin. Meetings are more productive in larger cities because the higher number of experts participating in each auction generates more intense competition for jobs, so that $b(k)$ increases with N for all k (Lemma 4).

3.2 *Sorting of the best experts*

Online Appendix G provides an alternative model with a time constraint, in which an expert's cost of searching for an entrepreneur to meet with is constant at c, and does not decrease with E. However, experts are able to find and meet only with the entrepreneurs for whom their knowledge is most valuable.[26] As in the model above, experts receive entrepreneur-specific draws, and they know the value of their knowledge for each entrepreneur that they meet. The model with sorting differs in that experts can use this information to sort into more productive meetings.

In such a model with a time constraint and constant search, meeting and work time costs, there are no increasing returns coming directly from the matching function (extensive margin). A larger pool of entrepreneurs is only desirable because it enhances experts' ability to find entrepreneurs for whom their knowledge is highly valuable. As the number of entrepreneurs tends to infinity, experts choose to meet only with entrepreneurs for whom their knowledge reaches its maximum value. In this sense, increasing returns arise because in larger cities,

[26]In the terminology of the search-theoretic literature, such a model is a directed search model, while the model presented before is a random search model. See Rogerson *et al.* (2005) for a review of that literature.

entrepreneurs meet experts better suited to their needs.[27] This sorting is another explanation for the higher productivity of workers in larger cities, beyond the higher number of meetings and heightened competition for jobs that are the sources of increasing returns in Section 2 and in the model with a time constraint presented above.

4. Migration into an open city

This section extends the closed city model of Section 2 to consider a country in which experts and entrepreneurs can choose to migrate to an open city. Open city models add an entry cost to the model of Section 2. In what follows, experts decide to migrate *before* learning their type, so there is no sorting of the best experts into the city (at the end of this section I discuss a model with sorting, and I solve such a model in online Appendix H.) The main result of this section is the existence of a unique locally stable spatial equilibrium city whose size, composition and knowledge markets are all inefficient. The auction framework suggests a tax policy to raise a city's productivity both by attracting more experts and by enhancing their incentives to transfer knowledge for free.

There is a large number of potential experts and entrepreneurs in the countryside, whose utility is normalized to 0. As before, $N = E + X$ is the city's population, and $z = E/X$ represents the city's composition. It is useful to express the number of entrepreneurs and experts in the city as a function of its population and composition, as $E = Nz/(z+1)$ and $X = N/(z+1)$. The job allocation mechanism (a first-price auction) and the production technology are as in the closed city model of Section 2. I introduce a cost of urban living C, in units of the good, which can be interpreted as housing rents or as a cost of driving to the city center to meet and work. C is a continuously differentiable, convex and increasing function of a city's population, with $C(0) = 0$, $C' > 0$ and $C'' > 0$

[27] In online Appendix G, I show that expected per capita production as city size tends to infinity (perfect sorting) is always larger than expected per capita production when the time constraint just binds. This demonstrates the presence of some increasing returns to city size over the range of populations for which the time constraint binds.

for all $N \geq 0$, and with $\lim_{N \to 0} C'(N)$ finite.[28] C is the same for experts and entrepreneurs and it covers the costs of rent plus travel to all meetings and work. An expert incurs C before learning her type, consistent with the idea that an expert's knowledge has a different value to different entrepreneurs, and that a meeting with an entrepreneur is necessary for an expert to discover the value of her knowledge to his particular needs.[29]

4.1 *Optimal city*

The optimal city maximizes per capita production minus the congestion cost (recall that utility is linear in the amount of the good consumed.) In the optimal city, each entrepreneur must use the knowledge of all experts in the city, so total expected production is $Y = \frac{XE}{2} = \frac{N^2 z}{2(z+1)^2}$. The optimal city then has a scale (N) and composition (z) that maximizes:

$$\max_{N,z} U(N,z) = \max_{N,z} \frac{Y}{N} - C(N) = \max_{N,z} \frac{Nz}{2(z+1)^2} - C(N). \tag{15}$$

Setting $\frac{\partial U}{\partial z} = 0$, one finds that $z = 1$ at any stationary point of U. So the optimal city features an equal number of entrepreneurs and experts, because of a symmetric matching function that generates more meetings per capita when $E = X$. Setting $\frac{\partial U}{\partial N} = 0$ and plugging $z = 1$ leads to $C'(N) = 1/8$ which defines optimal city size.[30]

[28] One way to derive such a cost function from a structural model is to add a congestion cost to a standard residential location model. In this model, workers consume land and a good. The cost of urban living $C(N)$ is equal to land rent plus the cost of a commute to a central business district to meet and work. Define cost of commute for someone who lives at distance d from the CBD as $\tau d N^\alpha$ (the N^α term is non-standard and represents a road congestion cost, which is necessary to obtain $C'' > 0$.). If land use is fixed at 1 unit, in a spatial equilibrium all variation in rent comes from the commute cost, and the derivative of rent with respect to distance from CBD must be $r'(d) = -\tau N^\alpha$. Assuming that land has no opportunity cost, its value outside of the city is 0 and the land rent function is $r(d) = -\tau N^\alpha (b - d)$, where b is the boundary of the city (note that $r(b) = 0$.) The city could be linear or circular. If linear, then it must be that $N = b$ for the land market to clear (recall that everyone consumes one unit of land.) So $C(N) = r(d) + \tau N^\alpha = \tau N^{1+\alpha}$, which satisfies all the assumption in the proposition.

[29] This intuition is formalized in the assumption that $corr(k_{ij}, k_{ih}) = 0$, for all $j,h \in \{1,...,E\}$, i.e. experts are not 'good' or 'bad' for every entrepreneur, but rather their knowledge has a different value to different entrepreneurs.

[30] $z = 1$ and $C'(N) = 1/8$ is the only stationary point of $U(z,N)$ and it is easy to show that it is a local maximum, with positive utility if $C(N)$ is not too large. To assess whether this point is a global maximum over all the positive real values of z and N, note that U equals 0 when N equals 0, and U becomes negative when either z or N tends to infinity or z equals 0.

4.2 Spatial equilibrium

Equilibrium city size and composition is determined by free migration of experts and entrepreneurs from the country-side.[31] An equilibrium must have three properties: first utility in the city and in the country side is equalized, second the equilibrium must be locally stable, and third experts must behave optimally in the knowledge transfer game.

When experts choose to migrate to a city before learning their type, the (sunk) cost of living does not affect their bid, or their decision to meet with all entrepreneurs. The same would be true if there was a fixed communication cost to each meeting, incurred before learning one's type. As a result, the benefits of living in the city stay exactly as in Section 2. Therefore, one can use equation (5) to obtain the expected utility of an expert who migrates to a city with population N and composition z:

$$U_X(N,z) = \frac{E}{X(X+1)} - C(N) = \frac{z(z+1)}{N+(z+1)} - C(N), \tag{16}$$

and equation (6) to obtain the expected utility of an entrepreneur:

$$U_E(N,z) = \frac{(X-1)}{2} - C(N) = \frac{N-(z+1)}{2(z+1)} - C(N). \tag{17}$$

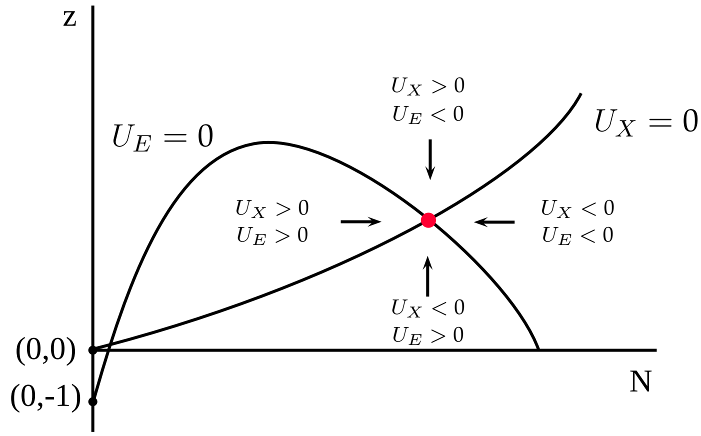
Experts and entrepreneurs only migrate if their expected utility in the city is positive, so the free-entry spatial equilibrium condition is:

$$\begin{aligned} U_E(N,z) &= 0 \\ U_X(N,z) &= 0. \end{aligned} \tag{18}$$

The easiest way to demonstrate the existence of a unique stable spatial equilibrium is to represent the indifference curves $U_E(N,z) = 0$ and $U_X(N,z) = 0$ in the (N,z) space. From equation (16), it is immediate that $\frac{\partial U_X}{\partial N} < 0$ and $\frac{\partial U_X}{\partial z} > 0$, for all $z > 0$ and $N > 0$, and therefore the indifference curve of an expert slopes upward from the origin (recall that $C(0) = 0$, so $U_X(0,0) = 0$). To determine the shape of an entrepreneur's indifference curve, I use equation (17), and isolate z

[31] Assuming costless individual migration to cities is standard and follows the seminal work of Henderson (1974). Helsley and Strange (2014) analyze the impact of costless individual migration in a system of cities model with many worker types.

Figure 1: Unique stable equilibrium in an open city



from $\frac{N-(z+1)}{2(z+1)} - C(N) = 0$ to find:

$$z = \frac{N}{2C(N)+1} - 1 \tag{19}$$

$$\frac{\partial z}{\partial N} = \frac{2C(N) + 1 - 2NC'(N)}{4(C(N) + 0.5)^2}. \tag{20}$$

From equation (20), the indifference curve of an entrepreneur slopes up from $(0, -1)$, which satisfies equation (19), then down.[32] Figure 1 represents both indifference curves.

From Figure 1, a locally stable equilibrium exists if the peak of an entrepreneur's indifference curve lies above the indifference curve of an expert. A sufficient condition for existence is that C takes small enough values, meaning that if $C = \tau\widetilde{C}$, a stable equilibrium exists for τ small enough (the following proposition includes a formal statement of this condition.) For instance, if $C = \tau N^2$, then it is easy to show that a stable equilibrium exists if $\tau < 0.02$.

The equilibrium is locally stable because experts and entrepreneurs move into or out of the city when their expected utility from such a move is positive. The

[32] To see this, note that the denominator of (20) is always positive. The limit of the numerator as $N \to 0$ is also positive, because $C(0) = 0$ and $\lim_{N\to 0} C'(N)$ is finite. As C is continuously differentiable, the indifference curve must slope up for small N. For N large enough, $\frac{\partial z}{\partial N}$ becomes negative, because $C' > 0$ and $C'' > 0$ imply that the negative term $-2NC'(N)$ grows faster than the positive term $2C(N)$ (the derivative of $NC'(N)$ is $NC''(N) + C'(N)$, and the derivative of $C(N)$ is $C'(N)$, so the assumption that $C' > 0$ and $C'' > 0$ for all $N > 0$ implies that $NC''(N) + C'(N) > C'(N)$, for all $N > 0$.)

four arrows of motion in Figure 1 illustrate that through such migration, the city returns to equilibrium after any small perturbation away from it. When both $U_X > 0$ and $U_E > 0$, N must increase, because both experts and entrepreneurs benefit from moving into the city. When $U_X < 0$ and $U_E < 0$, N must decrease, because both experts and entrepreneurs benefit from moving out of the city. When $U_X < 0$ and $U_E > 0$, z must increase, because entrepreneurs gain from moving into the city while experts gain from moving out of the city. By a similar reasoning, z decreases when $U_X > 0$ and $U_E < 0$. I summarize the above results in the following proposition:

Proposition 3 *Suppose that experts move into an open city before learning their types, that the expected utility of experts and entrepreneurs is given by equation* (16) *and* (17)*, that the cost of urban living is* $C = \tau\widetilde{C}$*, where* $\widetilde{C}(0) = 0$*,* $\widetilde{C}' > 0$ *and* $\widetilde{C}'' > 0$ *for all* N*,* $\lim_{N\to 0}\widetilde{C}'(N)$ *is finite, and that* E *and* X *can be any positive real numbers. Then there is a value* τ^* *such that for all* $\tau \in (0,\tau^*)$ *there exist a unique locally stable spatial equilibrium in which city composition* z *and city size* N *satisfy* $U_E(N,z) = U_X(N,z) = 0$*.*

Proof In Appendix A.[33] □

4.3 Sources of inefficiency in equilibrium

The equilibrium city differs from the optimal city due to three sources of inefficiency; in size, in composition and in knowledge transfers.

Size The classic result from Henderson (1974) is that cities are too large in equilibrium. In my model, there also exist out-of-equilibrium cities with smaller population than in equilibrium, and in which both experts and entrepreneurs are better-off. Figure 1 features areas with smaller N in which $U_X > 0$ and $U_E > 0$, instead of $U_X = U_E = 0$ as in equilibrium. Excess migration is a standard result in a self-organizing equilibrium, because agents do not internalize the negative congestion externality that they impose on others, and migrate until the benefits of city living exactly offset its costs. However, the next subsection shows that if

[33] If E and X must be integers, then there may not be any stable equilibrium even if all other conditions of Proposition 3 are satisfied.

C does not increase too fast, then the optimal city in my model is *larger* than the equilibrium one, because equilibrium composition is inefficient.[34]

Composition The work of Helsley and Strange (2014) on coagglomeration highlights the pervasiveness of inefficiencies in city composition. In a context with different industries and worker types, they argue that "individual migration is a weak instrument for promoting efficiency." To find equilibrium composition in my model, note that equilibrium requires that $z^{eq} = \{z : U_E = U_X\}$. Using equation (16) and (17), z^{eq} can be defined implicitly as:

$$N = \sqrt{(z^{eq}+1)^2(2z^{eq}+1)}.$$

This expression shows that z^{eq} grows like $N^{2/3}$, and the ratio of entrepreneurs to experts is larger than its optimal value of 1 for all $N > 2$. The ratio z^{eq} increases with city size, because in equilibrium the payoff of experts decreases with the number of other experts through tougher competition, so the number of entrepreneurs must increase even more to maintain experts' utility constant. This imbalance in the ratio of entrepreneurs to experts in equilibrium worsens as population increases, which reduces the number of meetings per capita. Therefore, increasing returns in production are weaker under self-organized migration than in a city with optimal composition. This implies that if the cost of urban living does not increase too fast, then the optimal city is larger than the equilibrium one.[35] This result contrasts with the standard case in Henderson (1974), and even with the result in Helsley and Strange (2014), in which the composition inefficiency reinforces the scale inefficiency. Ultimately, the number of experts in equilibrium is inefficiently low because they provide a positive externality without internalizing its benefits.

Knowledge transfers Finally, knowledge transfers are inefficient in the equilibrium city. Section 2 shows that experts do not transfer all their knowledge to entrepreneurs at the meeting stage of the game, although it would be efficient to

[34] Note that even if the optimal city is larger than the equilibrium one, there still exist cities with smaller population and higher utility than in equilibrium, as shown in Figure 1.

[35] If C increases fast, for instance if $C(N) = 0.5N^2$, then the optimal city is smaller than the equilibrium city, as in the canonical case.

do so. Knowledge spillovers, like any positive externality, create benefits that are not internalized, hence the inefficient provision of knowledge.

4.4 *Policy analysis*

A combination of two policies can achieve the optimal city. First, subsidizing the work of the best experts can lead to efficiency in composition and knowledge transfers. Second, restricting entry into the city can generate efficient city size.

A subsidy to an expert's work amounts to offering a wage top-up w. In equilibrium, the top-up always goes to the best experts, who win the knowledge auctions. Appendix B shows that in a first-price knowledge auction with a wage top-up w, the knowledge bid of an expert increases to $b(k,X,w) = ((X-1)/X)k + w$. If w is large enough, then all experts transfer the entire value of their knowledge during any meeting.[36] In this case, knowledge transfers are efficient. The subsidy can then be increased until city composition reaches its efficient level of $z = 1$.[37] Finally, migration in the city must be restricted to ensure that utility is not driven down to 0 by excess migration.

An interesting question is whether entrepreneurs themselves have an incentive to implement this policy. In a standard first-price auction, an auctioneer cannot increase his payoff by subsidizing the winner, because the cost of the subsidy exactly offsets the increase in the winner's bid. A knowledge auction, however, is all-pay for an entrepreneur, who finds it optimal to set a wage top-up w large enough to ensure that all types of experts transfer the entire value of their knowledge during the meeting stage (see Appendix B). Subsidizing the work of experts is a good policy because it incites every expert to transfer more knowledge, even those who fail to get the job. This result relates to Lazear and Rosen (1981) 'tournament' theory, which successfully explains the presence of oversized wages at the top of a firm's hierarchy, as a mean to motivate workers in the lower ranks.

[36]This solution assumes that the budget constraint of an expert never binds. That is, the total bid of an expert is always $b(k,X,w) = \frac{X-1}{X}k + w$. If an expert bids more than the value of her entire knowledge, then the difference is deducted from her wage should she win the job, or is just a separate payment to the entrepreneur.

[37]Appendix B shows that a subsidy of $1/X$ is sufficient to ensure that experts transfer all their knowledge. Using equation (16) and (17), it is easy to show that for all $N \geq 6$, the subsidy w must be larger than $1/X$ to equalize the utility of entrepreneurs and experts. Therefore, for $N \geq 6$, attaining efficient composition requires an additional subsidy beyond that necessary to make knowledge transfers efficient.

So entrepreneurs have incentives to implement a policy that makes knowledge transfers efficient, but unless they can coordinate they have no incentives to offer a subsidy large enough to insure optimal composition.[38]

Land developers or city governments can achieve the optimal city if they have the policy tools described above i.e. the power to subsidize the best experts and to restrict migration into the city. The implementation of both policies seems beyond the capabilities of land developers or local governments in the United States,[39] but it is noteworthy that the location decision of experts has become a current public policy issue. Florida (2002), for instance, argues that cities should strive to attract members of the 'creative class'. Florida recommends raising the value of urban living for the creative class, by subsidizing amenities that they like more than other people do. Such a policy is reasonable if creative types, like experts in the model, provide positive externalities.[40]

4.5 *Sorting by types*

Online Appendix H solves an open city model in which experts choose to migrate *after* learning their type. In this setting, the knowledge auction becomes a first-price auction with an endogenous entry cost. An analysis of the migration decision of experts reveals a trade-off between the quality and the quantity of experts migrating to the city. There are two main findings. First, only the best experts sort into the city. This sorting by skills is consistent with empirical evidence from Combes, Duranton, and Gobillon (2008) that more productive workers choose to live in larger cities. Second, an exogenous increase in the expected cost of urban living generates an increase in the average quality of experts living in the city, and a decrease in their expected number.

A unique feature of this model with sorting is that aggregate production depends directly on the cost of urban living, because a large entry cost decreases

[38]In Appendix B, entrepreneurs take the number of experts as given and do not take into account the effect of their subsidy on experts' entry. This is a good approximation when the number of entrepreneurs is large.

[39]See Helsley and Strange (2014) for a discussion of the role of land developers in a model with size and composition inefficiencies.

[40]In the real world, in which most experts already live in different cities, place-based policies designed to attract experts to a particular city have general equilibrium effects which impact their incidence (see Kline (2010) for a recent theoretical discussion.) Subsidies designed to enhance incentives for knowledge transfers, however, would still produce efficiency gains.

experts' willingness to transfer knowledge for free. Given that the cost of entry depends on the number of city inhabitants, knowledge bids do not necessarily increase with the number of experts competing for entry into the city. However, one can show that if the cost of living is low enough, then there are increasing returns in production to the number of experts in the country.[41]

5. Discussion

5.1 Interpretation of auction framework

Auction theory is relevant to knowledge transfers in cities because its main tenets have a plausible interpretation within the context of urban working and networking environments.

First, an auction implies that a free transfer is a bid, rather than a signal, a reciprocal transfer, or something else. In competitive urban environments, individuals care about recognition as more knowledgeable than their rivals for jobs, and a bid will often capture the motivation behind knowledge transfers better than a purely reciprocal exchange. This may explain why studies find reciprocity to be less important in industries featuring the strongest agglomeration economies. While extensions of the signaling model of Spence (1973) can explain voluntary knowledge disclosure for firms (e.g. Bhattacharya and Ritter 1983), the relative complexity of signaling games lessens their relevance to informal and semi-formal interactions, and makes them less amenable to modeling competitive

[41] In this model with sorting, the number of potential experts in the country-side matters, and it is not assumed to be infinite. There is little empirical evidence of increasing returns to population at the country level, but the model suggests that even negligible returns to the number of potential experts can be magnified into sizable returns to the number of city experts, that are observed in practice. This happens because experts staying in the countryside are not productive, while competition spurs both higher bids and sharper sorting of experts moving into the city. In a dynamic setting with many cities, Rossi-Hansberg and Wright (2007) show that a balanced growth path at the country level is consistent with increasing returns at the city level.

environments.[42]

Second, an auction implies that the value of an expert's knowledge corresponds to his valuation for a job. This condition holds if more knowledgeable experts, who perform better at work, earn larger rewards. Most industries feature some degree of performance pay, more so in agglomerated sectors like technology, professional services (finance, law, marketing, etc) and consulting. I argue in the next subsection that such sectors provide the clearest real-world counterparts to my model.

Third, an auction implies that the number of experts competing for a job affects the size of their knowledge bid. Intuitively, more intense competition for jobs should improve people's disposition towards free knowledge transfer. Empirical studies consistently find that bids in first-price auctions increase with the number of other bidders (Kagel, 1995). In the context of my model, this prediction really depends on the notion that one cannot be paid for knowledge already transferred for free, i.e. that knowledge is expropriable. In this case, there are clear incentives to transfer just enough knowledge to outbid all other experts, and the number of competing experts matters. In theory, expropriability is just a corollary of Arrow's property that knowledge needs to be possessed for its value to be assessed (Anton and Yao, 2002). Empirically, expropriability is hard to verify. In some industries, ethical concerns, social norms, regulations or non-disclosure agreements supercede entrepreneurs' pecuniary interest in expropriating experts.[43]

[42] In a basic auction model, individuals with higher valuation for a good (in this paper, a job) bid more. Signaling games, however, generally feature multiple equilibria, as the cost of sending a given message may be lower for high types, but the informativeness of this message depends on whether low types imitate it or not. Moreover, auction theory's straightforward predictions on the relationship between the number of bidders and the size of their bids, which are important to this paper on agglomeration, have no parallel in the signaling literature, which almost always feature one sender. One recent exception is from Barraquer and Tan (2012), who study competition in a signaling model with multiple senders. They show that when the number of senders is large enough, the only equilibrium satisfying a selection criterion is that in which all agents send the most informative message.

[43] There is certainly much anecdotal evidence, for instance, that designers providing free designs in the hope of winning a contest are paid below market rates. For a discussion see the article by Michelle Goodman *When to work for nothing* in the New York Times 'Shifting Careers' blog, November 9, 2008 (`http://shiftingcareers.blogs.nytimes.com/2008/11/09/when-to-work-for-free/`, retrieved 10 August 2013).

5.2 *Industry examples*

The model intends to expose the general motivation behind many informal or semi-formal urban interactions. The building blocks of the model, however, are more readily apparent in industries featuring a sharp distinction between holders and users of knowledge, and between a free knowledge transfer stage and an employment stage. Examples abound in 'creative' occupations like advertising, architecture, graphic design, finance, or in job interview situations, in which the model's 'bid' is often called a 'pitch'. Studying how these industries cope with free knowledge transfers allows to determine whether the strategic incentives of the agents in the model have parallels in the real-world. In the absence of hard data, the most supportive evidence for the theory comes from industries in which free knowledge transfers are most controversial, and therefore better documented. Free transfers appear especially unfair in the presence of communication costs, which, as I show in online Appendix H, can drive some experts completely out of the city.

Graphic design is such an industry, in which free transfers – called 'spec(ulative) work' or 'working on spec' – are deemed so problematic that some professional associations explicitly prohibit their members from providing free designs.[44] While a design or a logo does not possess all the strong properties of knowledge assumed in the model – designs are not necessarily freely reproducible, and there is a design cost – the bans highlight the incentives of experts to give knowledge for free in a competition for jobs.

In advertising and marketing, free knowledge transfers are somewhat less controversial, perhaps because of the lower cost of creating a marketing strategy or a slogan for a product. Unlike a design, a marketing advice can often be used productively even if it comes from an advisor who did not win a contract. In this case, free transfers not only generate better matches, but also more learning, as

[44] The Society of Graphic Designers of Canada, for instance, prohibits its members from providing free designs, or from entering into a design contest in which only the winner is remunerated (http://www.gdc.net/business/ethics_and_professional_practice/articles/186.php, retrieved 10 August 2013). In the United States, the largest such professional organization, AIGA, only goes so far as stating that: "AIGA believes that professional designers should be compensated fairly for their work [...]. To that end, AIGA strongly encourages designers to enter into client projects with full engagement to show the value of their creative endeavor, and to be aware of all potential risks before entering into speculative work." (http://www.aiga.org/position-spec-work/, retrieved 10 August 2013)

in the model.[45] This being said, there are also vocal opponents to free knowledge transfers within the advertising community. Win Without Pitching, a consulting firm for advertising agencies, voices the grievances of many agencies. An extract from The Win Without Pitching Manifesto (Enns, 2010) reads as follow: *"The forces of the creative professions are aligned against the artist. These forces pressure him to give his work away for free as a means of proving his worthiness of the assignment. Clients demand it. Advertising agencies and design firms resign themselves to it".* Indeed, competitive pressures to work for free can impose a significant burden on small players in big cities. From an efficiency perspective, one has to worry that such competition discourages individuals from acquiring knowledge, from actively developing creative ideas, or from incurring the cost of moving to a city.[46]

Finally, there is little evidence that uncompensated knowledge transfers are contentious in consulting industries, and among professionals such as lawyers, accountants, financial advisors and many other providers of business services.[47] Free knowledge transfers are part of standard networking practices. An example from the finance and banking industry is the mergers and acquisitions' (M&A) pitch, through which a firm specializing in M&A services provides a candidate firm with free information about the benefits of merging with, or acquiring another firm. Another example is 'stock pitching', an exercise to which expert practitioners have given sports-like qualities. Indeed, stock pitch competitions are popular with commerce students. Lawyers are often legally prevented from pitching free ideas to individuals or firms (to prevent 'ambulance chasing'), but most large firms have an in-house lawyer who can legally receive a pitch from

[45]Of course, experts often consider a situation unfair precisely when an entrepreneur profits from a free advice without offering a job.

[46]These issues are mostly beyond the scope of the paper, but online Appendix H shows that there are always benefits from competition – increasing returns in production to the number of experts – when the entry cost in the city is low enough.

[47]An interesting case of a free knowledge transfer in finance that *was* controversial comes from 'The Big Short', a book on the 2007-2008 financial crisis and the preceding housing bubble (Lewis, 2010) . The book tells the tale of Michael Burry, a fund manager who shares his ideas with potential investors on how to short the housing market and benefit from an eventual housing bust. Burry's idea, in retrospect, was worth billions. As it turned out, many of these potential investors did enter the trade Burry described, but never invested in Burry's fund (his guess is that they hired another 'expert', Goldman Sachs, to help). To quote Burry from the book: *"If I describe something it sounds compelling and people think they can do it themselves [...] if I don't describe it enough, it sounds scary and binary and I can't raise the capital".* This shows both Arrow's property and free reproducibility at work.

another lawyer hoping to land a contract, and such free advices are relatively common (Asher, 2004). Generally, consulting industries tend to share the model's main features: knowledge is (almost) freely reproducible, and it is impossible to take back knowledge transferred for free. Also, there are often obvious benefits from hiring one consultant, ideally the best, while getting knowledge from many. For instance, an entrepreneur needs at least one lawyer to sign legal papers, or one broker to trade stocks, but there is almost unlimited potential for legal actions to take or stocks to buy.

By and large, the entrepreneurs versus experts set-up is representative of the general motivation behind urban networking. Casual meetings and professional encounters are often opportunities to tell others what and how much one knows, and to learn about and from others' knowledge. These are the essential ingredients of successful networking; of a process that can lead to a job offer directly as in the model, or indirectly through name-dropping by others, or even to recognition as a valuable partner in a reciprocal relationship. In the model, experts strive to display their knowledge because of competition for entry into a contractual relationship with an entrepreneur, but competition for entry into reciprocal relationships likely fosters very similar incentives for knowledge transfers. These incentives find clear real-world manifestations not only in the business services described above, but also in high-tech industries and clusters like Silicon Valley, which are likely the environments with the strongest agglomeration economies. In these milieus, a reputation for expertise carries a high premium, and entrepreneurs must be aware of who knows what, and of how good they are. As a result, workers in high-tech clusters experience frequent job changes, and initially accept lower wages (Freedman, 2008) while hoping for a lucrative contract after establishing their reputation.

A potentially important obstacle to the productive use of knowledge transferred during meetings is the signature of a non-disclosure agreement (NDA). One expects NDAS to be largely absent from the kind of informal and semi-formal settings which are the model's closest real-world counterparts. Gault and von Hippel (2009) document the prevalence of such agreements in more formal settings, among Canadian manufacturing firms that have developed process innovations. Remarkably, they find that about half of these firms do not protect their innovation, which frequently ends up used by competitors and suppliers.

Anecdotal evidence also suggests that investors being pitched a startup idea will generally not sign an NDA, for fear of future litigation.[48]

5.3 *Sorting and knowledge diffusion*

Sections 3 and 4 briefly discuss extensions of the basic model featuring sorting of the best experts into meetings (these models are fully derived in the online appendix.) This sorting, motivated either by a time constraint in Section 3 or an entry cost in Section 4, provides entrepreneurs with better matches and can play an important role in explaining the production advantage of cities (Combes *et al.*, 2008).

The knowledge auction with an entry cost or a time constraint uncovers important trade-offs between the frequency and the quality of urban interactions. The downtowns of large cities are expensive places to live in, and face-to-face interactions are often costlier than other modes of communication. The impact of entry costs on the quality of participants in urban meetings is reminiscent of Storper and Venables (2004) quip about face-to-face interactions, that 'the medium *is* the message'. In a knowledge auction the message itself matters, but there are benefits from high entry costs, which encourage stricter sorting of the best experts into meetings. Among mechanisms for screening valuable communication partners, the leading alternative to voluntary sorting by willingness to pay is the design of reputation systems. Intuitively, reputation formation mechanisms become essential when communication costs are low. For instance, such mechanisms are key prerequisite for productive use of the Internet. In general, sorting and reputation formation are not mutually exclusive and can be part of the same process. If experts in the model were able to develop a reputation for being knowledgeable, they would be even better disposed towards free knowledge transfers while they establish their reputation.

Of course, lower communication costs also have advantages. Online Appendix H shows that a reduction in the cost of meeting can directly increases the size of experts' knowledge transfers. This demonstrates how low communication costs

[48] Michael Marasco, an expert on innovation and entrepreneurship from Northwestern University, puts it this way: "Venture capitalists and angel investors will laugh at you if you present them with a nondisclosure agreement [...]" Quote retrieved on 8 August 2014 from `http://www.chicagobusiness.com/article/20120811/ISSUE02/308119997/`.

can enhance incentives to disseminate knowledge. Importantly, such knowledge diffusion, which is the focus of the paper, is often a precondition to knowledge creation.[49] While many authors have rightly emphasized the salience of social norms and diversity for policies aiming at faster knowledge growth,[50] lowering the costs and augmenting the rewards from learning about others' ideas are also obvious targets.

6. Conclusion

Urban environments offer opportunities to learn and network, which take center stage in recent thinking about the purpose and future of cities. Starting from the key properties of knowledge as an input in production, I model knowledge transfers as bids in first-price auctions for jobs. The knowledge auction illustrates why knowledge sometimes 'spills' in non-market interactions before being bought and sold in markets. Endogenous agglomeration economies arise because of the greater number of meeting partners in urban areas, and heightened competition for jobs that enhances incentives for knowledge transfers. The model is too stylized to offer clear-cut policy recommendations, and it ignores the impact of reciprocity and diversity. However, it can inform discussions and empirical work on the origin of knowledge spillovers, and on how the cost of meeting and the number of potential competitors and partners affects an individual's decision to move to a city and transfer her knowledge.

[49]Online Appendix D captures knowledge creation in a rudimentary way, by introducing complementarities between the knowledge of different experts.

[50] Saxenian's (1994) comparative analysis of the technology clusters in Silicon Valley and Route 128 highlights social norms favoring knowledge flows as drivers of Silicon Valley's success. A theoretical literature emphasizes the importance of knowledge heterogeneity, see for instance Berliant, Reed III, and Wang (2006). Empirical studies also explore the impact of diversity on group decision making, for instance, West and Dellana (2009) find that both ability diversity and cognitive diversity reduce group decision errors.

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Appendix A. Proofs

Proof of Lemma 2

I first show that $c'(E) < 0$ implies $S'(E) > 0$. Using $E = zX$, I can express equation (7) as a function of E, and differentiate it implicitly to characterize $S'(E)$:

$$c'(E)S(E) + (c(E) + c(1))S'(E) - c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\left(\ln\left(1 - \frac{S(E)}{E}\right) + \frac{S(E) - S'(E)E}{E - S(E)}\right) = 0,$$

which is equivalent to:

$$\begin{aligned} & S'(E)\left(c(E) + c(1) + c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\frac{E}{E - S(E)}\right) \\ = \quad & c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\left(\ln\left(1 - \frac{S(E)}{E}\right) + \frac{S(E)}{E - S(E)}\right) - c'(E)S(E). \end{aligned} \tag{A1}$$

A necessary and sufficient condition for $S'(E) > 0$ is that:

$$c'(E) < \frac{1}{S(E)}\left[c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\left(\ln\left(1 - \frac{S(E)}{E}\right) + \frac{S(E)}{E - S(E)}\right)\right].$$

Using the inequality $\ln(x) > 1 - \frac{1}{x}$ for $0 < x < 1$, it follows that:

$$c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\left(\ln\left(1 - \frac{S(E)}{E}\right) + \frac{S(E)}{E - S(E)}\right) > 0. \tag{A2}$$

Hence a sufficient condition for $S'(E) > 0$ is that $c'(E) < 0$.

I now show that $c'(E) < \frac{-2}{E^2}$ implies $S'(E) > \left(\frac{S(E)}{E}\right)^2$. From equation (A1) it follows that $S'(E) > \left(\frac{S(E)}{E}\right)^2$ if and only if:

$$\frac{c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\left(\ln\left(1 - \frac{S(E)}{E}\right) + \frac{S(E)}{E - S(E)}\right) - c'(E)S(E)}{\left(c(E) + c(1) + c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\frac{E}{E - S(E)}\right)} > \left(\frac{S(E)}{E}\right)^2. \tag{A3}$$

Using equation (A2) again, a sufficient condition for (A3) to hold is that:

$$\frac{-c'(E)S(E)}{c(E) + c(1) + c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z}\frac{E}{E - S(E)}} > \left(\frac{S(E)}{E}\right)^2. \tag{A4}$$

or equivalently:

$$-c'(E) > \frac{S(E)}{E^2}\left(c(E) + c(1) + c(1)\left(1 - \frac{S(E)}{E}\right)^{E/z} \frac{E}{E - S(E)}\right). \qquad \text{(A5)}$$

Using the time constraint in equation (7), I can rewrite equation (A5) as:

$$-c'(E)E^2 > 1 + c(1)S\left(1 - \frac{S(E)}{E}\right)^{E/z-1} - c(1)z\left(1 - \left(1 - \frac{S(E)}{E}\right)^{E/z}\right). \qquad \text{(A6)}$$

To conclude that $c'(E) < \frac{-2}{E^2}$ is a sufficient condition for (A6) to hold, note that the third term on the right-hand side in equation (A6) is negative because $0 < S(E)/E < 1$, and that the second term is smaller than 1, because the time constraint implies that $c(1)S < 1$ and $\left(1 - \frac{S(E)}{E}\right)^{E/z-1} < 1$ because $0 < S(E)/E < 1$ and $E/z - 1 > 1$ for all $X > 1$.

Proof of Lemma 4

Before proving Lemma 4, it is useful to establish a preliminary lemma from Klenke and Mattner (2010). The lemma states conditions under which binomial distributions exhibit the monotone likelihood ratio property.

Lemma 5 *Let $\leq_{mlr}$ denote the monotone likelihood ratio property. That is, for two probability distributions P and Q, we have that $P \leq_{mlr} Q$ if their respective probability distribution functions p and q are such that $x \mapsto \frac{q(x)}{p(x)}$ is monotone nondecreasing. Let $B(n,p)$ be a binomial distribution for n trials with probability of success p. Then we have that $B_{n_1,p_1} \leq_{mlr} B_{n_2,p_2}$ if and only if $p_1 = 0$ or:*

$$n_1 \leq n_2 \text{ and } \frac{n_1 p_1}{1 - p_1} \leq \frac{n_2 p_2}{1 - p_2}.$$

Proof See Klenke and Mattner (2010), Section 1.2. □

I now prove Lemma 4, and show that the bidding function b is increasing in the number of experts X (or equivalently in city size N) for all $k \in (0,1]$. First consider the bidding function in equation (11), and note that $w_a(k,X) \in (0,1)$ for all $a \in \{1,...,X\}$ and that $\sum_{a=1}^{X} w_a(k,X) = 1$, so that $w_a(k,X)$ is a probability. In this sense, the bidding function $b(k) = \sum_{a=1}^{X} w_a(k,X)\frac{a-1}{a}k$ is just the expected value of a lottery with payoffs $\frac{a-1}{a}k$, so we can write:

$$b(k,X) = \mathbb{E}_{w(k,X)}\left(\frac{a-1}{a}\right)k.$$

$\frac{a-1}{a}$ is increasing in a, so to prove that b is increasing in X it suffices to show that the probability distribution induced by $w_a(k,X)$ is stochastically increasing in X - in the sense of first-order stochastic dominance - for each $k \in (0,1)$. A sufficient condition for strict first-order stochastic dominance is the strict monotone likelihood ratio property, i.e. that:

$$a \mapsto \frac{w_a(k,X+1)}{w_a(k,X)}$$

is monotone increasing.

To show that this condition holds, note that:

$$\frac{w_a(k,X+1)}{w_a(k,X)} = \frac{p(a,X+1)}{p(a,X)} \frac{\sum_{i=1}^{X} k^{i-1} p(i,X+1)}{\sum_{i=1}^{X+1} k^{i-1} p(i,X)}$$

is increasing in a if and only if $\frac{p(a,X+1)}{p(a,X)}$ is increasing in a, i.e. if the binomial $B_{X,q(X)}$ displays the strict monotone likelihood ratio property.[51] Using the conditions in Lemma 5, we find that $B_{X,q(X)} <_{mlr} B_{X+1,q(X+1)}$ if:[52]

$$X\frac{q(X)}{1-q(X)} < (X+1)\frac{q(X+1)}{1-q(X+1)}. \tag{A7}$$

The remainder of the proof is just to show that inequality (A7) holds. Given that $q(X) = S(E)/E$ and that $E/z = X$, I can write $X\frac{q(X)}{1-q(X)} = \frac{S(E)E}{z(E-S(E))}$ to find:

$$\frac{\partial \frac{S(E)E}{z(E-S(E))}}{\partial E} = \frac{S'(E) - \left(\frac{S}{E}\right)^2}{zE^2(E-S(E))^2}. \tag{A8}$$

So the proposition follows if $S'(E) > \left(\frac{S}{E}\right)^2$, because it implies that $\frac{\partial X \frac{q(X)}{1-q(X)}}{\partial X} > 0$ so that equation (A7) holds. As shown in Lemma 2, a sufficient condition for $S'(E) > \left(\frac{S}{E}\right)^2$ is $c'(E) < \frac{-2}{E^2}$.

[51] Recall that $s(a,X)$ is the probability of a bidders generated by the binomial distribution $B_{X,q(X)}$, and $\frac{s(a,X+1)}{s(a,X)}$ increases in a if and only if $\frac{p(a,X+1)}{p(a,X)}$ increases in a, because:

$$\frac{s(a,X+1)}{s(a,X)} = \frac{(X+1)q(X+1)}{Xq(X)} \frac{p(a,X+1)}{p(a,X)}.$$

[52] If the inequalities in Lemma 5 are strict, then the lemma yields the strict monotone likelihood ratio property.

Proof of Proposition 2

The proof relies on two results. First, the bidding function is increasing in N (established in Lemma 4). Second, the probability distribution governing the number of bidders a induced by $s(.,X)$ is stochastically increasing in X (or equivalently, in N).

I first rewrite expected total production from equation (14) in the following way:

$$\begin{aligned} Y(N) &= E\sum_{a=1}^{X} s(a,X)\frac{a}{a+1} + \int_0^1 b(y,X)\left(EQ - \sum_{a=1}^{X} as(a,X)y^{a-1}\right)\mathrm{d}y \\ &= E\sum_{a=1}^{X} s(a,X)\frac{a}{a+1} + \int_0^1 b(y,X)\sum_{a=1}^{X} as(a,X)(E-y^{a-1})\mathrm{d}y, \qquad \text{(A9)} \end{aligned}$$

where the second equality uses $Q = \sum_{a=1}^{X} as(a,X)$. It is possible to express equation (A9) using expectations with respect to the probability distribution $s(.,X)$ as follows:

$$Y(N) = E\mathbb{E}_{s(.,X)}\left[\frac{a}{a+1}\right] + \int_0^1 b(y,X)\mathbb{E}_{s(.,X)}[a(E-y^{a-1})]\mathrm{d}y.$$

To characterize this equation, we first need to show that the probability distribution induced by $s(.,X)$ is stochastically increasing in X, in the sense of first-order stochastic dominance. Exactly as in the proof of Lemma 4, a sufficient condition is that the sequence $\{s(.,X)\}_{X\in\mathbb{N}_{>0}}$ satisfies the strict monotone likelihood ratio property, which was established in the proof of Lemma 4 using the condition $c'(E) < \frac{-2}{E^2}$.

It is now possible to complete the proof using the definition of increasing returns, and to show that $Y(\alpha N) > \alpha Y(N)$ for all $\alpha > 1$. So I write:

$$\begin{aligned} Y(\alpha N) &= \alpha E\mathbb{E}_{s(.,\alpha X)}\left[\frac{a}{a+1}\right] + \int_0^1 b(y,\alpha X)\mathbb{E}_{s(.,\alpha X)}[a(\alpha E - y^{a-1})]\mathrm{d}y \\ &> \alpha E\mathbb{E}_{s(.,X)}\left[\frac{a}{a+1}\right] + \int_0^1 b(y,\alpha X)\mathbb{E}_{s(.,X)}[\alpha a(E-y^{a-1})]\mathrm{d}y, \qquad \text{(A10)} \end{aligned}$$

where the second inequality in equation (A10) follows because the distribution induced by s is stochastically increasing in X and the expectations with respect to s are defined over an sequence of increasing outcomes. Using the fact that $b(.,X)$

is also increasing in X we obtain:

$$\begin{aligned} Y(\alpha N) &> \alpha E\mathbb{E}_{s(.,X)}\left[\frac{a}{a+1}\right] + \int_0^1 b(y,X)\mathbb{E}_{s(.,X)}[\alpha a(E-y^{a-1})]\mathrm{d}y \\ &> \alpha\left(E\mathbb{E}_{s(.,X)}\left[\frac{a}{a+1}\right] + \int_0^1 b(y,X)\mathbb{E}_{s(.,X)}[a(E-y^{a-1})]\mathrm{d}y\right) \\ &= \alpha Y(N), \end{aligned} \tag{A11}$$

which concludes the proof.

Proof of Proposition 3

In the text, I showed that there exist a unique stable equilibrium if and only if the $U_X(N,z) = 0$ and $U_E(N,z) = 0$ curves cross for some $(N,z) \in \mathbb{R}^2_{>0}$. I now provide a condition on C under which the stable equilibrium exists. I show that if the cost of urban living $C = \tau\widetilde{C}$ is small enough (i.e. if τ is small enough), then it must be that $U_X = U_E = 0$ for some $(N,z) \in \mathbb{R}^2_{>0}$.

The first step is to find N as a function of z such that $U_X = U_E$. Equating equation (16) and (17) I find:

$$N = \sqrt{(z+1)^2(2z+1)}. \tag{A12}$$

Recall that $U_X = \frac{z(z+1)}{N+(z+1)} - C(N)$, so in equilibrium it must be that:

$$\frac{z(z+1)}{\sqrt{(z+1)^2(2z+1)} + (z+1)} - \tau\widetilde{C}\left(\sqrt{(z+1)^2(2z+1)}\right) = 0, \tag{A13}$$

where I replaced N in equation (16) by its equilibrium value from equation (A12). Note that for large values of z, the positive term on the left-hand side of equation (A13) grows like $z^{1/2}$, while the negative term grows at least like $z^{3/2}$ (the input of $\widetilde{C}$ grows like $z^{3/2}$ and the second derivative of $\widetilde{C}$ is positive.) It follows that for large enough z, the negative term in equation (A13) grows faster than the positive term. I conclude that if τ is small enough, then there is a value $z^{eq} > 0$ such that the equality in equation (A13) holds. From equation (A12), there also exist a value of $N^{eq} > 0$ such that $U_X(N^{eq},z^{eq}) = U_E(N^{eq},z^{eq}) = 0$.

Appendix B. Auctions with transfer payments: offering a wage top-up

This appendix extends the basic model of Section 2 to let entrepreneurs make transfer payments to experts, outside of the wage setting mechanism specified in Section 2. I restrict attention to transfers going to experts who win a knowledge auction, which are equivalent to a wage top-up, denoted by w. I show that an entrepreneur gains from setting $w > 0$, and that any w large enough to ensure that all types of experts bid the entire value of their knowledge is optimal from the perspective of an entrepreneur.

Suppose that expert i meets an entrepreneur j offering a wage top-up w_j. In a symmetric equilibrium, this expert submits a bid $b_i \geq 0$ that solves:

$$\max_{b_{ij}} prob\left(b_{ij} > \max_{l \in \{1,\dots,X\}\setminus\{i\}} b(k_{lj}, X, w_j)\right) \times (k_{ij} + w_j - b_{ij})|_{b(k_{ij})=b_{ij}}. \quad \text{(A14)}$$

This expression is similar to equation (2) governing experts' behavior in the basic first-price auction of Section 2, aside from the extra w_j in the payoff. Equation (A14) implicitly assumes that an expert does not face a binding budget constraint. That is, whenever an expert bids more than the entire value of her knowledge, any part of the bid that exceeds her knowledge value is deducted from her wage should she get the job. It is straightforward to prove that the bidding function becomes $b(k, X, w) = \frac{X-1}{X}k + w$ (I removed subscripts to improve legibility).

The optimal w is that which maximizes U_E, the expected utility of an entrepreneur, which is equal to:

$$U_E = X\left(\int_0^{wX} y dy + \int_{wX}^1 \left(\frac{X-1}{X}y + w\right) dy\right) + \int_0^{wX}\left(w - \frac{y}{X}\right) X y^{X-1} dy - w. \quad \text{(A15)}$$

The first of the three terms on the right-hand side of equation (A15) is the expected value of the total knowledge transferred by all experts during meetings. The first integral in the parenthesis of the first term is the expected value of the bids for experts with types $((X-1)/X)k + w > k$ who bid the entire value of their knowledge at the meeting stage.[53] The second integral in the parenthesis is the expected value of the bids for experts who have enough knowledge to transfer

[53] To compute the boundaries of the integral, note that wX solves $((X-1)/X)k + w = k$.

$((X-1)/X)k+w$ at the meeting stage. The second term in equation (A15) is the expected deduction from the wage of an expert who won the auction but bid all her knowledge at the meeting stage, and owes the difference $((X-1)/X)k+w-k=w-\frac{k}{X}$ to the entrepreneur.[54] The third term is minus the wage top-up. Note that equation (A15) is only correct over the domain $w \in [0{,}1/X]$. For w larger that $1/X$, all types of experts bid their entire knowledge at the meeting stage.

To find the value of w that maximizes U_E, I solve the integrals in equation (A14) to obtain:

$$U_E = \frac{X-1}{2} + w(X-1) - w^2\frac{X^2}{2} + \frac{(wX)^{X+1}}{X(X+1)},$$

and I take a derivative with respect to w to find:

$$\frac{\partial U_E}{\partial w} = (X-1) - wX^2 + (wX)^X.$$

This derivative is positive at $w=0$, and it decreases with w until it reaches 0 at $w=1/X$. Therefore, over the domain $w \in [0{,}1/X]$, the expected utility of an entrepreneur reaches a maximum at $w=1/X$, which is a wage top-up just high enough to ensure that all types of experts bid the entire value of their knowledge. At $w=1/X$, the expected payoff of an entrepreneur is $\frac{X}{2}-\frac{1}{1+X}$, and it is easy to show that it stays constant for all $w>1/X$. I conclude that an entrepreneur chooses a wage-top up large enough to motivate all types of experts to transfer all their knowledge for free at the meeting stage.

[54]Recall that $f(k_{(X,X)}) = Xk_{(X,X)}^{X-1}$ is the probability distribution function of the highest order statistic from X draws of the $i.i.d.$ $U[0{,}1]$ distribution.